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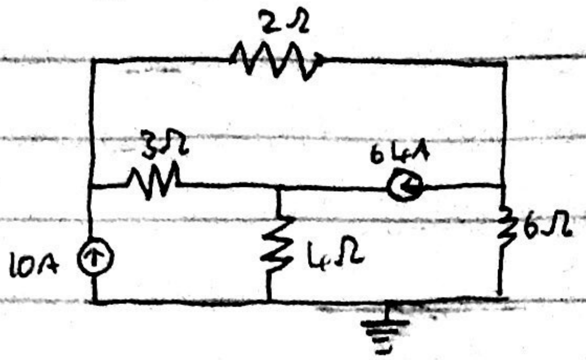
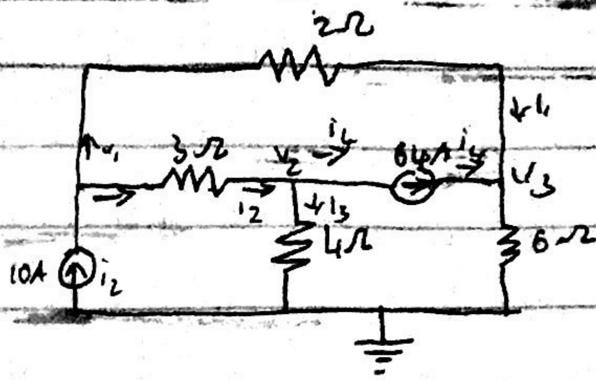
17/ENG05/038

Mechatronics Engineering

ENG 322





At node 1, KCL:

$$10 = i_1 + i_2 \Rightarrow 10 = \frac{V_1 - V_2}{2} + \frac{V_1 - V_2}{3}$$

$$\Rightarrow 60 = 3(V_1 - V_3) + 2(V_1 - V_2)$$

$$60 = 3V_1 - 3V_3 + 2V_1 - 2V_2$$

$$60 = 5V_1 - 2V_2 - 3V_3 \quad \cdots (i)$$

At Node 2, KCL:

$$i_2 = i_3 + 64$$

$$64 = i_2 - i_3$$

$$64 = \frac{V_1 - V_2}{3} - \frac{V_2 - 0}{4}$$

$$768 = 4(V_1 - V_2) - 3(V_2 - 0)$$

$$768 = 4V_1 - 4V_2 - 3V_2$$

$768 = 4V_1 - 7V_2$ .... (ii)

At Node 3, KCL;

$64 + i_1 = i_3$

$64 = i_3 - i_1$

$$64 = \frac{V_3 - 0}{6} - \frac{V_1 - V_3}{2}$$

$384 = V_3 - 3(V_1 - V_3)$

$384 = -3V_1 + 4V_3$ .... (iii)

Using Crammer's Rule

$5V_1 - 2V_2 - 3V_3 = 60$ .... (i)

$4V_1 - 7V_2 - 0V_3 = 768$ .... (ii)

$-3V_1 + 0V_2 + 4V_3 = 384$ .... (iii)

In matrix Respresentation

$$\begin{bmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 0 & 4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 768 \\ 384 \end{bmatrix}$$

$$V_1 = \frac{\Delta_1}{\Delta}, \quad V_2 = \frac{\Delta_2}{\Delta}, \quad V_3 = \frac{\Delta_3}{\Delta}$$

Where $\Delta = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{vmatrix} 5 & -2 & -3 \\ 4 & -7 & 0 \\ -3 & 6 & 4 \end{vmatrix} \end{array}$

$= 5(-28) + 2(16) - 3(21)$

$= -140 + 32 + 63$

$= -45$

$\Delta_1 = \begin{vmatrix} 60 & -2 & -3 \\ 768 & -7 & 0 \\ 384 & 0 & 4 \end{vmatrix} = 60(-28) - 768(-8) + 384(-21)$

$= -1680 + 6144 - 8064$

$= -3600$

$\therefore v_1 = \frac{\Delta_1}{\Delta} = \frac{-3600}{-45} = 80V$

For v_2 : $\Delta_2 = \begin{array}{c} + \\ - \\ + \end{array} \begin{vmatrix} 5 & 60 & -3 \\ 4 & 768 & 0 \\ -3 & 384 & 4 \end{vmatrix}$

$= 5(768 \times 4) - 4[(60 \times 4) - (384 - 3)] - 3[0 - (768 - 3)]$

$= 2880$

$\therefore v_2 = \frac{\Delta_2}{\Delta} = \frac{2880}{-45} = -64V$

For v_3; $\begin{array}{c} + \\ - \\ + \end{array} \begin{vmatrix} 5 & -2 & 60 \\ 4 & -7 & 768 \\ -3 & 0 & 384 \end{vmatrix}$

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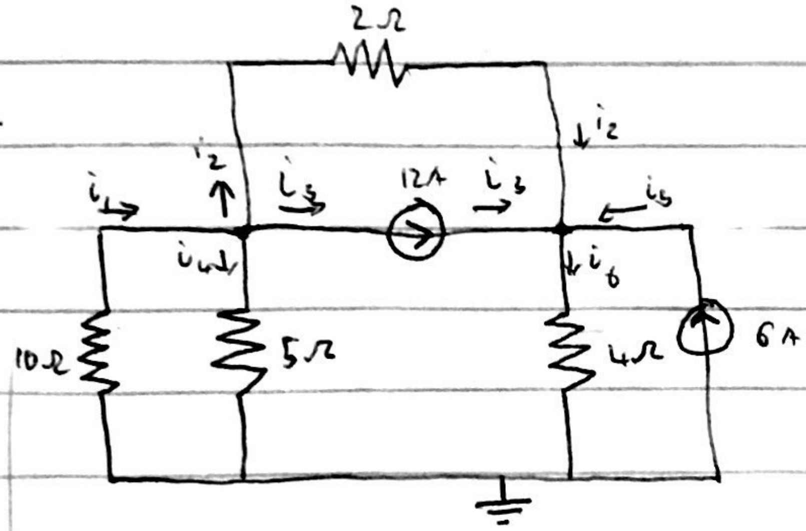
$= 5[(-7\times384)-0] - 4[(-2\times384)-0] - 3[(-2\times768)-(-7\times60)]$

$= -7020$

$\therefore V_3 = \frac{\Delta_3}{\Delta} = \frac{-7020}{-45} = 156V$

Hence $V_1 = 80V$, $V_2 = -64V$, $V_3 = 156V$.

2.



At Node 1; KCL

$i_1 = i_2 + i_3 + i_4$

$\frac{V_0 - V_1}{10} = \frac{V_1 - V_2}{2} + 12 + \frac{V_1 - V_0}{5}$

$0 - V_1 = 5(V_1 - V_2) + 120 + 2(V_1 - 0)$

$-V_1 = 5V_1 - 5V_2 + 120 + 2V_1$

$120 = -8V_1 + 5V_2 \quad \cdots\cdots (i)$

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At Node 2

$i_3 + i_2 + i_5 = i_4$

$12 + \frac{V_1 - V_2}{2} + 6 = \frac{V_2 - 0}{4}$

$96 + 4(V_1 - V_2) + 48 = 2V_2$

$144 + 4V_1 - 4V_2 = 2V_2$

$144 = -4V_1 + 6V_2 \quad \ldots\ldots \text{(ii)}$

Using Elimination Method

$120 = -8V_1 + 5V_2 \quad \ldots\ldots \text{(i)} \quad \times -4$

$144 = -4V_1 + 6V_2 \quad \ldots\ldots \text{(ii)} \quad \times -8$

$-480 = 32V_1 - 20V_2 \quad \ldots\ldots \text{(iii)}$

$-1152 = 32V_1 - 48V_2 \quad \ldots\ldots \text{(iv)}$

(iv) - (iii) $\quad -672 = 0 - 28V_2$

$V_2 = \frac{-672}{28}$

$= 24V$

Subs. $V_2 = 24$ in eqn (ii)

$144 = -4V_1 + 6V_2$

$V_1 = \frac{144 - 6(24)}{-4}$

$= 0$

$\therefore V_1 = 0V, \quad V_2 = 24V$

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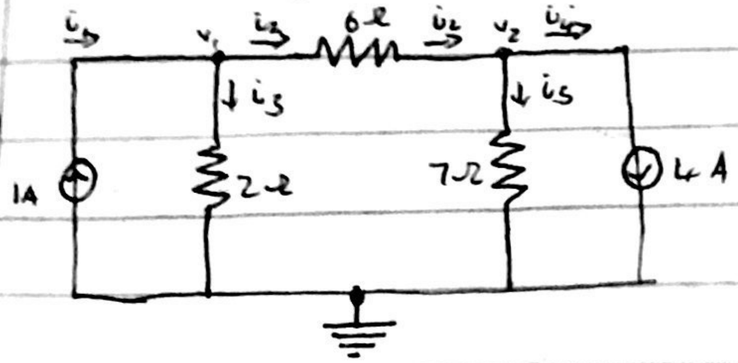
$i_1 = 0A$, $i_2 = 0A$, $i_3 = 6A$, $i_4 = -12A$

ii)



At Node 1

$$i_1 = i_2 + i_3$$

$$1 = \frac{v_1 - v_2}{6} + \frac{v_1}{2}$$

$$6 = v_1 - v_2 + 3v_1$$

$$6 = 4v_1 - v_2 \quad \cdots \text{(i)}$$

At Node 2

$$i_2 = i_4 + i_5$$

$$\frac{v_1 - v_2}{6} = 4 + \frac{v_2}{7}$$

$$7(v_1 - v_2) = 168 + 6v_2$$

$$168 = 7v_1 - 7v_2 - 6v_2$$

$$168 = 7v_1 - 13v_2 \quad \cdots \text{(ii)}$$

From eqn (i) ; $v_2 = 4v_1 - 6 \quad \cdots \text{(iii)}$

Subs. $v_2 = 4v_1 - 6$ in eqn (ii)

$$168 = 7V_1 - 13(4V_1 - 6)$$
$$168 = 7V_1 - 52V_1 + 78$$
$$90 = -45V_1$$
$$V_1 = -2V$$

Subs $V_1 = -2$ in eqn (1a)

$$V_2 = 4(-2) - 6$$
$$V_2 = -14V$$

$\therefore V_1 = -2V, \quad V_2 = -14V$

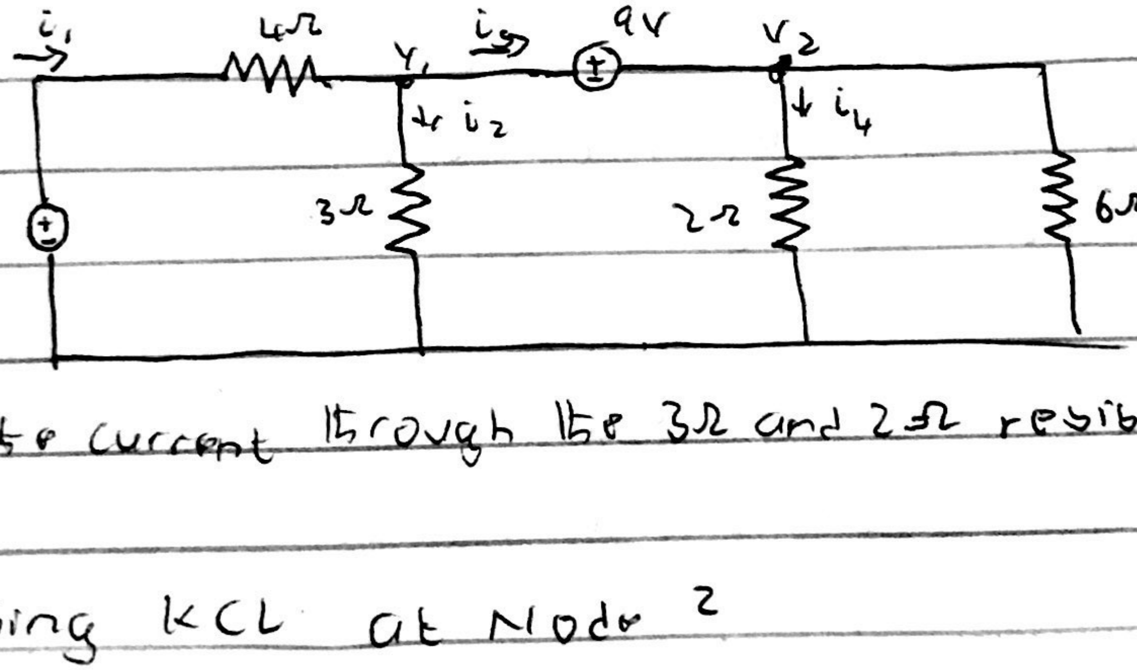
Current through the resistors,

$$i_2 = \frac{V_1 - V_2}{6} = \frac{-2+14}{6} = 2A$$

$$i_3 = V_1/2 = -2/2 = -1A$$

$$i_5 = V_2/7 = -14/7 = -2A$$

3.



Find the current through the 3Ω and 2Ω resistors.

Using KCL at Node 2

$$i_1 + i_2 + i_3 + i_4 = 0$$

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$$\frac{V_1 - 21}{4} + \frac{V_1}{3} + \frac{V_2}{6} + \frac{V_2}{2}$$

$$7V_1 + 8V_2 - 63 = 0 \quad \cdots (i)$$

Using KVL For Loop 1

$$-V_1 - 9 + V_2 = 0$$

$$-V_1 + V_2 = 9 \quad \cdots (ii)$$

$\cancel{7V_1}$

$$7V_1 + 8V_2 = 63 \quad \cdots (i)$$

$$-V_1 + V_2 = 9 \quad \cdots (ii)$$

From Eqn (ii) Let $V_2 = 9 + V_1$

Sub $V_2 = 9 + V_1$ in Eqn (i)

$$7V_1 + 8(9 + V_1) = 63$$

$$7V_1 + 72 + 8V_1 = 63$$

$$15V_1 = -9$$

$$V_1 = -0.6V$$

Sub $V_1 = -0.6$ in eqn (ii)

$$-(-0.6) + V_2 = 9$$

$$0.6 + V_2 = 9$$

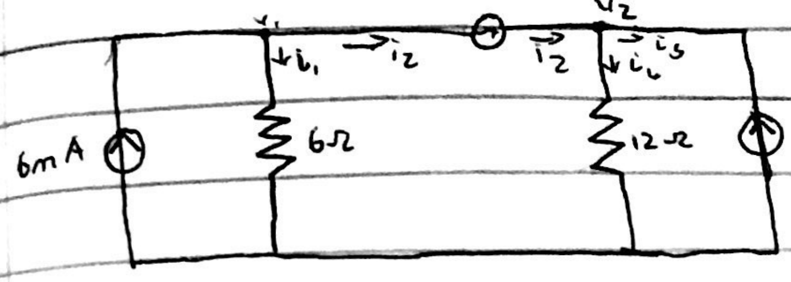
$$V_2 = 8.4V$$

$\therefore V_1 = -0.6V$ and $V_2 = 8.4V$

Current through the 3Ω resistor;

$$i_2 = \frac{v_2}{4} = \frac{8.4}{4} = 4.2A$$

4.



Find the node voltages and the currents through 6Ω and 12Ω resistor.

Let assume that $v_1 - v_2 = 6V \Rightarrow i_2$

At node 1; using KCL

$$6mA = i_1 + i_2$$

$$6mA = \frac{v_1 - 0}{6} + (v_1 - v_2)$$

$$36 = v_1 + 6(v_1 - v_2)$$

$$36 = v_1 + 6v_1 - 6v_2$$

$$36 = 7v_1 - 6v_2 \quad \cdots\cdots (i)$$

At node 2

$$i_2 = i_3 + i_4$$

$$v_1 - v_2 = 4mA + \frac{v_2 - 0}{12}$$

$$12(v_1 - v_2) = 48 + v_2$$

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$48 = 12V_1 - 12V_2 - V_2$

$48 = 12V_1 - 13V_2 \cdots (ii)$

Solving V_1 and V_2 simultaneously, we have

$V_1 = 9.5V$ and $V_2 = 5.1V$

$\therefore$ Current through the 6Ω resistor

$i_1 = \frac{V_1}{6} = \frac{9.5}{6} = 1.58A$; $I_2 = V_1 - V_2 = 9.5 - 5.1$

$= 4.4A$

$\therefore$ Current through the 12Ω resistor

$i_4 = \frac{V_2}{12} = \frac{5.1}{12} = \cancel{1.58A, i_2}\ 0.43A$

$\therefore V_1 = 9.5V, V_2 = 5.1V$

$i_1 = 1.58A$; $i_6 = 0.63A$

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